Engineering

PERTURBED EULER SCHEME FOR WEDGE-LIKE BODIES IN SUPERSONIC AND HYPERSONIC FLOW: ANALYSIS AND OPTIMIZATION

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Inviscid supersonic and hypersonic flows are formerly governed by Euler equations. Based on Lighthill's wedge perturbation concept [1], the uniform steady supersonic wedge flow is adopted as the base flow solution for the present perturbation scheme. Thus, a wedge-like profile with either concave or convex curvature can be approximated by the present perturbed formulation to the first order in profile slopes. The requirement is that the perturbed shock shape must remain attached at the apex of the wedge. The perturbed Euler equations can be recast in a single variable pressure formulation. When the pressure equation is transformed from the physical to a quasi-conical coordinate system, a similarity solution for the pressure is found. A class of power-law bodies can be derived with the similarity solution established. With the application of the Mellin transform, and its inverse, the present formulating can be generalized to account for a wider class of body shapes, yielding the pressure field and shock shape as part of the solution.

Similar analytical methods have been developed in the past dealing with the hypersonic end of the spectrum. Hui [2] used a similarity method to develop a solution that results in a logarithmic shock, where Cole and Aroesty [3] employed a similarity solution to develop a solution that results in an exponential shock. Since these solutions are limited to specific shapes and to the hypersonic small disturbance assumptions, a unified supersonic/hypersonic solution for more general shapes, such as the present, is desirable. To this effect, the present solution is compared and found to agree well with these hypersonic solutions for the corresponding bodies when very high Mach numbers are used. In addition, excellent agreements are found through various case comparisons with results obtained by Computational Fluid Dynamics (CFD). However, the present analytical solution can be solved with a relatively simple MATLAB code in around 20 seconds. On the other hand, the same case takes CFL3D, a NASA supported CFD code, around 30 minutes to solve, in addition to the tedious effort in grid generation. This clearly shows that an efficient optimization plan could be developed for the present solution, but not for the CFD approach.

Several different shape optimization problems can be considered depending on the objective functions and constraints. Specifically, the first problem to be considered will be the drag minimization for a non-lifting body, i.e. for which the upper and lower surfaces are symmetric. Minimum drag will be sought for a constant body thickness ratio and for a set Mach number and specific heat ratio. The perturbed body shape is to be represented as a third order polynomial by sections or as a power-law function. The design variables for the optimization are the coefficients in the shape perturbation functions. The next optimization problem will focus on lifting bodies, and will seek a maximum lift to drag ratio. The shape representation will be the same as the above problem, but the upper and lower bodies will no longer be symmetric.

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1. Lighthill, M.J., *The Flow behind a Stationary Shock*, Phil. Mag. Vol. 40,1949, 214-220.
2. Hui, W.H., *A Solution for Hypersonic Flow past Slender Bodies*, JFM, 1971, Vol 48, Pt 1, 23-31.
3. Cole, J.D., Aroesty, J, *Hypersonic Similarity Solutions for Airfoils Supporting Exponential Shock Waves*, AIAA J., 1970 Vol 8, No. 2, 308-315.